Control and stabilization of delayed systems

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Master Automatic control and systems

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Summery

Motivations:

- Most systems are nonlinear
- Delay complicates the system analysis
- Delay can lead to the system instability
- ...

The TS fuzzy model can be justified by:

- Its simplicity
- Uncertainties
- Its acceptable accuracy
- ....

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The system can be modeled as:

$$\begin{cases} {}^{C}D^{\alpha}x(t) = f(x(t), x(t-\tau(t)), u(t)),\ t \geq 0, \\ x(s) = \varphi(s),\ s \in [-\tau, 0] \end{cases}$$

$x(t) \in \Re^{n}$ the system state
$u(t) \in \Re^{m}$ the control vector
τ: the delay

Lemma 1

$$I(x,t) = \int_{a(t)}^{b(t)} f(x,t)\,dx \quad , \quad a(t) \;\; \& \;\; b(t) < \infty$$

with $a(t) \quad b(t)$ and $f(x,t)$

$$\frac{d(I(x,t))}{dt} = \frac{db(t)}{dt} f(b(t),t) - \frac{da(t)}{dt} f(a(t),t) + \int_{a(t)}^{b(t)} \frac{\partial f(x,t)}{\partial t}\,dx$$

Lemma 2

$$I(x,t) = \int_{a(t)}^{b(t)} f(x,t)\,dx \quad , \quad a(t) \;\; \& \;\; b(t) < \infty$$

with $a(t) \quad b(t)$ and $f(x,t)$

$$\frac{d(I(x,t))}{dt} = \frac{db(t)}{dt} f(b(t),t) - \frac{da(t)}{dt} f(a(t),t) + \int_{a(t)}^{b(t)} \frac{\partial f(x,t)}{\partial t}\,dx$$

Theorem

Let's consider:

$$\begin{cases} \Omega_{ii} < 0, \ (i = 1, 2, ...r), \\ \Omega_{ij} + \Omega_{ji} < 0, \ i < j, \ i, j = 1, 2, ...r \end{cases}$$

with:

$$\Omega_{ij} = \begin{bmatrix} PA_i + A_i^T P^T + PB_i K_j + K_j^T B_i^T P^T + Q & PA_{di} & A_i^T N^T + K_j^T B_i^T N^T & 0 \\ * & -(1-\mu)Q & A_{di}^T N^T & 0 \\ * & * & \tau^2 R - N - N^T & 0 \\ * & * & * & -R \end{bmatrix}$$

We consider: $u(t) = \sum_{i=1}^{r} h_i(\theta(t))[K_i x(t) + K_{di} x(t-\tau(t))]$

Theorem

$$\begin{cases} \bar{\Omega}_{ii} < 0, \; i = 1, 2, ...r \\ \bar{\Omega}_{ij} + \bar{\Omega}_{ji} < 0, \; i < j, \; i, j = 1, 2, ...r \end{cases}$$

where:

$$\bar{\Omega}_{ij} = \begin{bmatrix} A_i X + X^T A_i^T + B_i Y_j + Y_j^T B_i^T + \bar{Q} & A_{di} X + B_i Y_{dj} & \epsilon X^T A_i^T + \epsilon Y_j^T B_i^T & 0 \\ * & (1-\mu)\bar{Q} & \epsilon X^T A_{di}^T + \epsilon Y_{di}^T B_i^T & 0 \\ * & * & \tau^2 \bar{R} - \epsilon X - \epsilon X^T & 0 \\ * & * & * & -\bar{R} \end{bmatrix}$$

$K_j = Y_j X^{-1} \quad K_{dj} = Y_{dj} X^{-1} (j = 1, 2, ...r)$

System behavior without controller

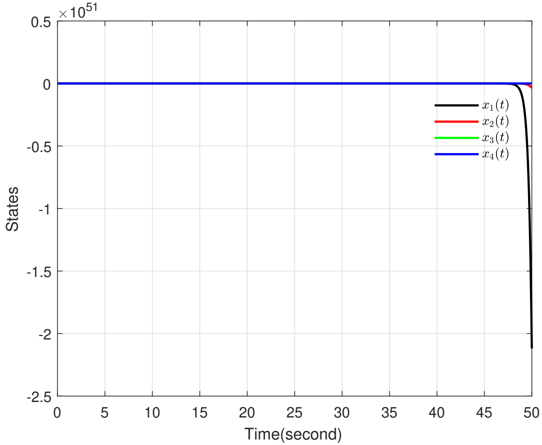

Figure 1: System state

System unstable

System behavior with controller

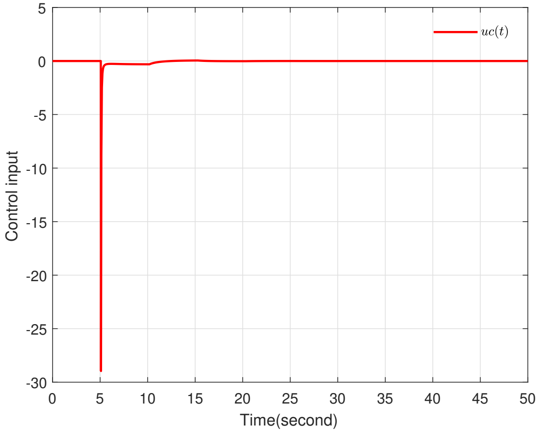

Figure 2: Control signal

System behavior with controller

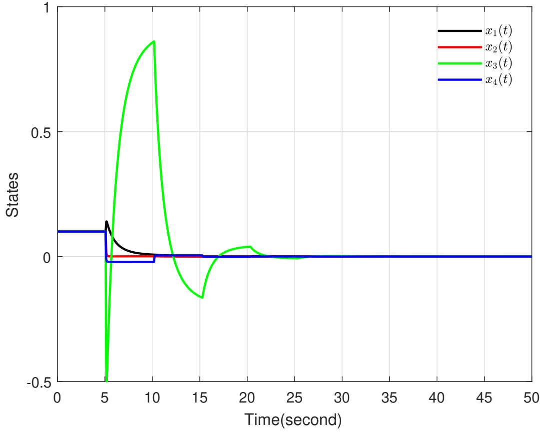

Figure 3: System state

The system is stable but it needs more enhancement

System behavior with the proposed controller

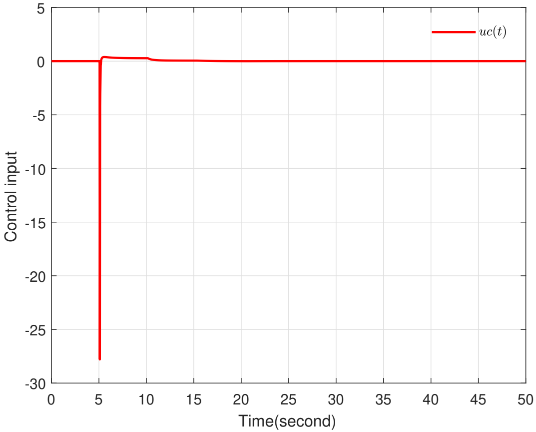

Figure 4: Proposed controller signal

System behavior with the proposed controller

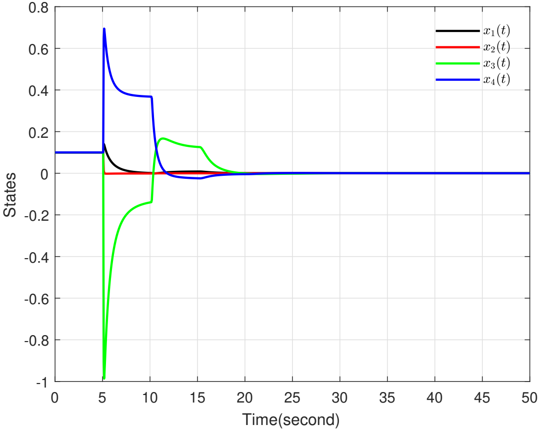

Figure 5: System state with the proposed controller

Stability + better performance

Quantification of the comparative study

	Classical controller	Proposed controller	Enhancement rate
Settling time	26	20	23 %
Pic to pic x_3	1.36	1.16	15 %
$\int_0^{ts} (x_3^2)\, dt$	2.5540	0.7771	70 %
$\int_0^{ts} (u^2)\, dt$	12.8476	7.1868	40 %

Table 1: Quantification of the comparative study

We remark that

- **Enhancement of the settling time of** 23%
- **Reduction of the control energy by** 40%
- **Overall enhancement by** 70%
- **Pic to pic reduction by** 15%

We conclude that:

Conclusion

- **Lyapunov method efficiency.**
- **Proposed controller leads to better performance.**
- **Delayed controller enhances the performance.**
- **Proposed approach allows reduction of the control energy.**

As perspectives we propose:

perspectives

- Perspective 1.
- Perspective 2.
- Perspective 3.